RENORMALIZABILITY AND THE MODEL INDEPENDENT OBSERVABLES FOR ABELIAN Z' SEARCH

A. V. Gulov * and V. V. Skalozub †
Dniepropetrovsk State University, Dniepropetrovsk, 320625 Ukraine
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The observables useful for the model independent search for signals of the abelian Z' in the processes $e^+e^- \to \bar{f}f$ are introduced. They are based on the renormalization group relations between the Z' couplings to the Standard Model particles developed recently and extend the variables suggested by Osland, Pankov and Paver. The bounds on the values of the observables at the center-of-mass energy $\sqrt{s} = 500$GeV are derived.

I. INTRODUCTION

The abelian Z'-boson with the mass much larger than the W-boson mass ($m_{Z'} \gg m_W$) is predicted by a number of extensions of the Standard Model (SM) of elementary particles [1]. At the current energies $\sim m_W$ the Z' is decoupled. It can be described by the model based on the effective gauge group $SU(2)_L \times U(1)_Y \times \tilde{U}(1)$ which is assumed to be a low energy remnant of some unknown underlying theory (GUT, for example). However, the Z' would be light enough to give the first signal in future experiments.

Due to the lower mass limit from the Tevatron, $m_{Z'} > \mathcal{O}(500)$GeV, only the 'indirect' Z' manifestations caused by virtual heavy states can be searched for at the energies of the present day accelerators. In general, one is not able to estimate the magnitude of the Z' signal because of the unknown couplings and the mass. However, numerous strategies to evidence manifestations of the Z' in experiments at high energy e^+e^- and hadronic colliders have been developed. The analysis can be done with or without assumptions on the specific underlying theory containing the Z'. Hence, there are model dependent and model independent variables allowing to detect the Z'.

One of the model independent approaches useful in searching for the Z' signal in the leptonic processes $e^+e^- \to \ell^+\ell^-$ has been proposed in Ref. [2]. The basic idea was to replace the standard observables, the total cross section σ_T and the forward-backward asymmetry A_{FB}, by the new set of variables defined as the differences of the cross sections integrated over suitable ranges of polar angle θ

$$\sigma_\pm \equiv \pm \int\limits_{\mp z^*}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta \mp \int\limits_{-1}^{\mp z^*} \frac{d\sigma}{d\cos\theta} d\cos\theta. \qquad (1)$$

Due to the SM values of the leptonic charges and the kinematic properties of the fermionic currents they have chosen the value $z^* = 2^{2/3} - 1 = 0.5874$ to make the leading order deviations from the SM predictions $\Delta\sigma_\pm$ dependent on the combinations $v^e_{Z'} v^\ell_{Z'} \pm a^e_{Z'} a^\ell_{Z'}$, where $v^f_{Z'}$ and $a^f_{Z'}$ parameterize the vector and the axial-vector coupling of the Z' to the fermion f. Therefore, assuming the lepton universality one obtains the sign definite observable $\Delta\sigma_+$.

Usually, the parameters describing at low energies the Z' coupling to the SM fermions (like $v^f_{Z'}$ and $a^f_{Z'}$) are assumed to be arbitrary numbers which must be fixed in experiments. However, this is not the case if the renormalizability of the underlying theory is taken into account. In Refs. [3], [4] it has been shown that, if one uses the principles of the renormalization group (RG) and the decoupling theorem the correlations between the parameters describing interactions of light particles with heavy virtual states of new physics beyond the SM, can be derived. Most important that the relations obtained, being the consequence of the renormalizability formulated in the framework of scattering in the external field, are independent of the specific underlying (GUT) model.

In Ref. [4] the method was applied to find signals of the heavy abelian Z' in the four-fermion scattering processes. It was found that the renormalizability of the theory is resulted in the following constraints on the Z' couplings to the SM fermions:

$$v^{f'}_{Z'} - a^{f'}_{Z'} = v^f_{Z'} - a^f_{Z'}, \quad a^f_{Z'} = I^f_3 Y_\phi, \qquad (2)$$

where f' denotes the iso-partner of f ($\ell' = \nu_\ell$, $\nu'_\ell = \ell$, $q'_d = q_u$, $q'_u = q_d$, where $\ell = e, \mu, \tau$ stands for leptons), I^f_3 is the third component of the weak isospin and Y_ϕ is the hypercharge parameterizing the Z' coupling to the SM scalar doublet. Since the parameters of various fermionic processes are appeared to be correlated, one could expect that it is possible to introduce the specific observables sensitive to the Z' manifestations. In the present paper we propose such the observables which generalize the variables $\sigma_\pm$ (1).

The content is as follows. In Sect. II the structure of neutral currents induced by the Z'-boson as well as the Z-Z' mixing is briefly discussed. Employing the relations (2) between the Z' parameters the optimal observables for searching for signals of the heavy abelian Z' are

*E-mail address: gulov@ff.dsu.dp.ua
†E-mail address: skalozub@ff.dsu.dp.ua

constructed in Sect. III. In Sect. IV the experimental bounds on the observables are predicted. The obtained results are summarized in Sect. V.

II. NEUTRAL CURRENTS AND VECTOR BOSON MIXING

Considering the interactions of the Z' with the SM particles, one has to conclude that at low energies, $E \ll m_{Z'}$, the renormalizable interactions are to be dominant. The terms of the non-renormalizable type (for example, $\sim (\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)\bar{\psi}\sigma_{\mu\nu}\psi)$, being generated at GUT (or some intermediate $\Lambda^{GUT} > \Lambda' > m_{Z'}$) mass scale, are suppressed by the factors $1/\Lambda^{GUT}$, $1/\Lambda'$ and can be neglected. Thus, the interaction of the Z' boson with the fermionic currents can be specified by the effective Lagrangian

$$\mathcal{L}_{NC} = eA_\mu J^\mu_A + g_Z Z_\mu J^\mu_Z + g_{Z'} Z'_\mu J^\mu_{Z'}, \tag{3}$$

where A, Z, Z' are the photon, the Z- and Z'-bosons, respectively, $e = \sqrt{4\pi\alpha}$, $g_Z = e/\sin\theta_W \cos\theta_W$, and $g_{Z'}$ stands for the $\tilde{U}(1)$ coupling constant. θ_W denotes the SM value of the Weinberg angle ($\tan\theta_W = g'/g$, where the charges g, g' correspond to the gauge groups $SU(2)_L$, $U(1)_Y$, respectively). The neutral currents can be parameterized as

$$J^\mu_V = \sum_f \bar{f}\gamma^\mu \left(v^f_V + a^f_V \gamma^5\right) f, \tag{4}$$

with $V \equiv A, Z, Z'$. The vector and the axial-vector couplings of the vector boson i to the fermion f are

$$v^f_A = Q_f, \quad a^f_A = 0, \tag{5}$$

$$\begin{aligned} v^f_Z &= \left(\frac{I^f_3}{2} - Q_f \sin^2\theta_W\right)\cos\theta_0 + \frac{g_{Z'}}{g_Z} Y^v_f \sin\theta_0, \\ a^f_Z &= -\frac{I^f_3}{2}\cos\theta_0 + \frac{g_{Z'}}{g_Z} Y^a_f \sin\theta_0, \end{aligned} \tag{6}$$

$$\begin{aligned} v^f_{Z'} &= Y^v_f \cos\theta_0 - \frac{g_Z}{g_{Z'}}\left(\frac{I^f_3}{2} - Q_f \sin^2\theta_W\right)\sin\theta_0, \\ a^f_{Z'} &= Y^a_f \cos\theta_0 + \frac{g_Z}{g_{Z'}}\frac{I^f_3}{2}\sin\theta_0, \end{aligned} \tag{7}$$

where Q_f is the fermion charge in the positron charge units. The constants Y^v_f and Y^a_f parameterize the vector and the axial-vector coupling of the fermion f to the $\tilde{U}(1)$ symmetry eigenstate, whereas θ_0 is the mixing angle relating the mass eigen states Z_μ, Z'_μ to the massive neutral components of the $SU(2)_L \times U(1)_Y$ and the $\tilde{U}(1)$ gauge fields, respectively. Its value can be determined from the relation [5]

$$\tan^2\theta_0 = \frac{m^2_W/\cos^2\theta_W - m^2_Z}{m^2_{Z'} - m^2_W/\cos^2\theta_W}. \tag{8}$$

Because of the mixing between the Z and Z' bosons the mass m_Z differs from the SM value $m_W/\cos\theta_W$ by the small quantity of order $m^2_W/m^2_{Z'}$ [4]

$$m^2_Z = \frac{m^2_W}{\cos^2\theta_W}\left(1 - \frac{4g^2_{Z'}Y^2_\phi}{g^2}\frac{m^2_W}{m^2_{Z'} - m^2_W/\cos^2\theta_W}\right), \tag{9}$$

and, as a consequence, the parameter $\rho \equiv m^2_W/m^2_Z \cos^2\theta_W > 1$. Therefore, the mixing angle θ_0 is also small $\theta_0 \simeq \tan\theta_0 \simeq \sin\theta_0 \sim m^2_W/m^2_{Z'}$.

The difference $m^2_Z - m^2_W/\cos^2\theta_W$ is negative and completely determined by the Z' coupling to the scalar doublet. Thus, constraints on the Z' interaction with the scalar field can be obtained by experimental detecting this observable:

$$\frac{g^2_{Z'}Y^2_\phi}{m^2_{Z'}} = \left(1 - \frac{m^2_Z\cos^2\theta_W}{m^2_W}\right)\frac{g^2}{4m^2_W} + O\left(\frac{m^4_W}{m^4_{Z'}}\right). \tag{10}$$

As it has been proven in Ref. [4], the following relations hold for the constants Y^v_f and Y^a_f

$$Y^L_{f'} = Y^L_f, \quad Y^a_f = I^f_3 Y_\phi, \tag{11}$$

where $Y^L_f \equiv Y^v_f - Y^a_f$, $Y^R_f \equiv Y^v_f + Y^a_f$, and Y_ϕ is the hypercharge parameterizing the coupling of the SM scalar doublet to the vector boson associated with the $\tilde{U}(1)$ symmetry. As it is noted in Sect. I, the notation f' stands for the iso-partner of f.

In fact, the relation (11) means that the Z' couplings to the SM axial currents have the universal absolute value, if a single light scalar doublet exists. Among the four values Y^v_f, Y^a_f, $Y^v_{f'}$, $Y^a_{f'}$ parameterizing interaction of the Z'-boson with the $SU(2)$ fermionic isodoublet only one is independent. The rest ones can be expressed through it and the hypercharge Y_ϕ of the Z' coupling to the SM scalar doublet. If the hypercharges are treated as unknown parameters these relations are to be taken into account in order to preserve the gauge symmetry [4]. The relations (11) also show that the fermion and the scalar sectors of the new physics are strongly connected. As a result, the couplings of the Z'-boson to the SM axial currents are completely determined by its interaction with the scalar fields. Therefore, one is able to predict the Z' coupling to the SM axial currents by measuring the ρ parameter. When the Z' does not interact with the scalar doublet, the Z-boson mass is to be identical to its SM value. In this case the Z' couplings to the axial currents are produced by loops and to be suppressed by the additional small factor $g^2/16\pi^2$.

III. THE OBSERVABLES

In the present section we consider the electron-positron annihilation into fermion pairs $e^+e^- \to V^* \to \bar{f}f$ for

energies $\sqrt{s} \sim 500$GeV. In this case all the fermions except for the t-quark can be treated as massless particles $m_f \sim 0$. The Z'-boson existence causes the deviations ($\sim m_{Z'}^{-2}$) of the cross section from its SM value:

$$\Delta \frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} - \frac{d\sigma_{SM}}{d\Omega} = \frac{\mathrm{Re}\left[T_{SM}^{*} \Delta T\right]}{32\pi s} + O(\frac{s^2}{m_{Z'}^4}), \quad (12)$$

with

$$T_{SM} = T_A + T_Z|_{\theta_0=0}\,, \quad \Delta T = T_{Z'} + \left.\frac{dT_Z}{d\theta_0}\right|_{\theta_0=0} \theta_0, \quad (13)$$

where T_V denotes the Born amplitude of the process $e^+e^- \to V^* \to \bar{f}f$ with the virtual V-boson ($V = A, Z, Z'$) state in the s-channel (the corresponding diagram is shown in Fig. 1).

The quantity $\Delta d\sigma/d\Omega$ can be calculated in the form

$$\Delta \frac{d\sigma}{d\Omega} = \frac{\alpha I_3^f N_f}{4\pi} \sum_{\lambda,\xi} \frac{g_{Z'}^2 \zeta_{\lambda\xi}^{ef}}{m_{Z'}^2} \left(|Q_f| + \chi(s)\left(\mathrm{sgn}\lambda - \varepsilon\right)\right.$$
$$\left.\times \left(\mathrm{sgn}\xi + |Q_f|(1-\varepsilon) - 1\right)\right)\left(z + \mathrm{sgn}\lambda\xi\right)^2, \quad (14)$$

where $N_f = 3$ for quarks and $N_f = 1$ for leptons, $\lambda, \xi = L, R$ denotes the fermion helicity states, $\chi(s) = \left(16 \sin^2\theta_W \cos^2\theta_W (1 - m_Z^2/s)\right)^{-1}$, $z \equiv \cos\theta$ (where θ is the angle between the incoming electron and the outgoing fermion), $\varepsilon \equiv 1 - 4\sin^2\theta_W \sim 0.08$, and

$$\zeta_{\lambda\xi}^{ef} \equiv Y_e^\lambda Y_f^\xi - \frac{m_W^2/\cos^2\theta_W}{s - m_Z^2} \left(Y_\phi Y_f^\xi \left(2\sin^2\theta_W - \delta_{\lambda,L}\right)\right.$$
$$\left. + 2 I_3^f Y_\phi Y_e^\lambda \left(-2|Q_f|\sin^2\theta_W + \delta_{\xi,L}\right)\right), \quad (15)$$

with $\delta_{\lambda,\xi} = 1$ when $\lambda = \xi$ and $\delta_{\lambda,\xi} = 0$ otherwise.

To discuss the physical effects caused by the relation (11), let us introduce the observable $\Delta\sigma(Z)$ defined as the difference of cross sections integrated in suitable ranges of $\cos\theta$

$$\sigma(Z) \equiv \int_Z^1 \frac{d\sigma}{dz} dz - \int_{-1}^Z \frac{d\sigma}{dz} dz \quad (16)$$

The value of Z will be chosen later. Actually, this observable is the generalized σ_+ of Ref. [2] ($\sigma_+ = \sigma(-z^*)$). The two conventionally used observables, the total cross section σ_T and the forward-backward asymmetry A_{FB}, can be obtained by special choice of Z ($\sigma_T = \sigma(-1)$, $A_{FB} = \sigma(0)/\sigma_T$). In fact, one can express $\sigma(Z)$ in terms of σ_T and A_{FB}

$$\sigma(Z) = \sigma_T \left(A_{FB}\left(1 - Z^2\right) - \frac{1}{4} Z\left(3 + Z^2\right)\right). \quad (17)$$

Owing to the relations (11) the quantity $\Delta\sigma(Z) \equiv \sigma(Z) - \sigma_{SM}(Z)$ can be written in the form

$$\Delta\sigma(Z) = \frac{\alpha N_f}{8} \frac{g_{Z'}^2}{m_{Z'}^2} \left(F_0^f(Z,s) Y_\phi^2 + 2F_1^f(Z,s) I_3^f Y_f^L Y_e^L\right.$$
$$\left. + 2F_2^f(Z,s) I_3^f Y_f^L Y_\phi + F_3^f(Z,s) Y_e^L Y_\phi\right). \quad (18)$$

The functions $F_i^f(Z,s)$ depend on the fermion type through the $|Q_f|$, only. In Figs. 2-4 they are shown as the functions of Z for $\sqrt{s} = 500$GeV. The leading contributions to $F_i^f(Z,s)$

$$F_0^f(Z,s) = -\frac{4}{3}|Q_f|\left(1 - Z - Z^2 - \frac{Z^3}{3}\right) + O\left(\varepsilon, \frac{m_Z^2}{s}\right),$$
$$F_1^f(Z,s) = \frac{4}{3}\left(1 - Z^2 - |Q_f|\left(3Z + Z^3\right)\right) + O\left(\varepsilon, \frac{m_Z^2}{s}\right),$$
$$F_2^f(Z,s) = -\frac{2}{3}\left(1 - Z^2\right) + \frac{2}{9}\left(3Z + Z^3\right) \times \left(4|Q_f| - 1\right) + O\left(\varepsilon, \frac{m_Z^2}{s}\right),$$
$$F_3^f(Z,s) = \frac{2}{3}|Q_f|\left(1 - 3Z - Z^2 - Z^3\right) + O\left(\varepsilon, \frac{m_Z^2}{s}\right) \quad (19)$$

are given by the Z' exchange diagram (the first term of Eq.(15)), since the contribution of the Z exchange diagram to ΔT (the second term of Eq.(15)) is suppressed by the factor m_Z^2/s.

From the Eqs. (19) one can see that the leading contributions to the leptonic factors F_1^ℓ, F_2^ℓ, F_3^ℓ are found to be proportional to the same polynomial in Z. This is the characteristic feature of the leptonic functions F_i^ℓ originated due to the kinematic properties of fermionic currents and the specific values of the SM leptonic charges. Therefore, it is possible to choose the value of $Z = Z^*$ which switches off three leptonic factors F_1^ℓ, F_2^ℓ, F_3^ℓ simultaneously. Moreover, the quark function F_3^q in lower order is proportional to the leptonic one and therefore is switched off, too. As is seen from Figs. 2-4, the appropriate value of Z^* is about ~ 0.3. By choosing this value of Z^* one can simplify Eq.(18). It is also follows from Eq. (18) that neglecting the factors F_1^ℓ, F_2^ℓ, F_3^ℓ one obtains the sign definite quantity $\Delta\sigma_\ell(Z^*)$.

Comparing the observable $\Delta\sigma_\ell(Z^*)$ with $\Delta\sigma_+ = \Delta\sigma_\ell(-0.5874)$ or $\Delta\sigma_- = -\Delta\sigma_\ell(0.5874)$ one can to conclude that the sign of the variables $\Delta\sigma_\pm$ is completely undetermined in the case of arbitrary leptonic couplings Y_ℓ^L. Therefore, in order to predict the sign of the observables $\Delta\sigma_\pm$ one has to assume the additional restriction such as the lepton universality.

Let the value of Z^* in Eq.(16) is determined from the relation

$$F_1^\ell(Z^*, s) = 0. \quad (20)$$

The solution $Z^*(s)$ is shown in Fig 5. As is seen, Z^* decreases from 0.3170 at $\sqrt{s} = 500$GeV to 0.3129 at $\sqrt{s} = 700$GeV. Table I demonstrates the corresponding behavior of the functions $F_i^f(Z^*, s)$. Since $F_i^f(Z^*, s)$ depend on the center-of-mass energy through the small quantity m_Z^2/s, the order of the shifts is about 3%. Therefore, in what follows the value of $\sqrt{s}$ is taken to be 500GeV.

Assuming $Y_e^L \sim Y_\ell^L \sim Y_\phi \sim Y_u^L \sim 1$, one can derive

$$\begin{aligned}
\Delta\sigma_\ell(Z^*) &= -0.10\frac{\alpha g_{Z'}^2 Y_\phi^2}{m_{Z'}^2}\left(1 + O(0.04)\right), \\
\Delta\sigma_{q_u}(Z^*) &= 1.98\Delta\sigma_\ell(Z^*) + 0.32\frac{\alpha g_{Z'}^2 Y_\phi}{2m_{Z'}^2} \\
&\quad\times\left(\left(Y_e^L/Y_\phi - 0.6\right)Y_{q_u}^L + O(0.07)\right), \\
\Delta\sigma_{q_d}(Z^*) &= 0.94\Delta\sigma_\ell(Z^*) - 0.32\frac{\alpha g_{Z'}^2 Y_\phi}{m_{Z'}^2} \\
&\quad\times\left(\left(Y_e^L/Y_\phi - 0.6\right)Y_{q_u}^L + O(0.08)\right), \qquad (21)
\end{aligned}$$

Hence it is seen, that the observable $\Delta\sigma_\ell(Z^*)$ is negative. Moreover, it can be written in terms of the ρ parameter using Eq.(10):

$$\Delta\sigma_\ell(Z^*) \simeq 0.10\frac{\alpha g^2(1-\rho)}{4m_W^2\rho} < 0. \qquad (22)$$

One also can construct the sign definite observable for quarks of the same generation. As it follows from Eq. (21),

$$\Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*) \simeq 2.45\Delta\sigma_\ell(Z^*) < 0. \qquad (23)$$

Hence it follows, that the values of $\Delta\sigma_{q_u}(Z^*)$ and $\Delta\sigma_{q_d}(Z^*)$ in the $\Delta\sigma_{q_u}(Z^*)$ – $\Delta\sigma_{q_d}(Z^*)$ plane have to be at the line crossing axes at the points $\Delta\sigma_{q_u}(Z^*) = 2.45\Delta\sigma_\ell(Z^*)$ and $\Delta\sigma_{q_d}(Z^*) = 4.9\Delta\sigma_\ell(Z^*)$, respectively. It also follows from Eq.(22) that the observable $\Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*)$ is negative.

Thus, the dependencies (2) between the Z' couplings to SM fermions allows to construct three negative valued observables, $\Delta\sigma_\ell(Z^*)$, $1-\rho$ and $\Delta\sigma_{q_u}(Z^*)+0.5\Delta\sigma_{q_d}(Z^*)$, which are correlated by Eqs. (22)-(23). These observables are the most general model independent ones which can be introduced without any assumptions such as the lepton or quark universality.

IV. THE EXPERIMENTAL CONSTRAINTS ON THE OBSERVABLES

The present day experimental data constrain the magnitude of the four-fermion contact interactions allowing to derive bounds on the Z' coupling to the axial currents and, consequently, on the observables introduced in the previous section. Our analysis is based on the data presented in Refs. [6] where the study of the experimental bounds on the lepton-quark four-fermion contact couplings has been performed. In general, the contributions of new physics beyond the SM to the considered therein processes (the atomic parity violation experiment as well as the electron-nucleus, muon-nucleus and ν_μ-nucleon scattering experiments) are described by 20 parameters, namely,

$$\eta_{\lambda\xi}^{\ell q} \equiv -\frac{g_{Z'}^2 Y_\ell^\lambda Y_q^\xi}{m_{Z'}^2}, \quad \eta_{L\xi}^{\nu_\mu q} \equiv -\frac{g_{Z'}^2 Y_{\nu_\mu}^L Y_q^\xi}{m_{Z'}^2}, \qquad (24)$$

where $\ell = e, \mu$; $q = u, d$ and $\lambda, \xi = L, R$. In order to reduce the number of independent $\eta_{\lambda\xi}^{\ell q}$ one usually assumes the $SU(2)_L$ invariance and the lepton universality. As a result, six variables (for example, $\eta_{LL}^{\ell u}$, $\eta_{LR}^{\ell u}$, $\eta_{LR}^{\ell d}$, $\eta_{RL}^{\ell u}$, $\eta_{RR}^{\ell u}$, $\eta_{RR}^{\ell d}$) can be chosen as the basis.

However, the number of independent $\eta_{\lambda\xi}^{\ell q}$ can be further decreased by employing the correlations (2). In this case it is useful to introduce the couplings $\eta_{AA}^{\ell q}$, $\eta_{LA}^{\ell q}$, $\eta_{AL}^{\ell q}$ parameterizing the four-fermion interactions between the left-handed and the axial-vector currents. These couplings are the linear combinations of the variables (24):

$$\begin{aligned}
\eta_{AA}^{\ell q} &\equiv \eta_{RR}^{\ell q} - \eta_{RL}^{\ell q} - \eta_{LR}^{\ell q} + \eta_{LL}^{\ell q}, \\
\eta_{LA}^{\ell q} &\equiv \eta_{LR}^{\ell q} - \eta_{LL}^{\ell q}, \\
\eta_{AL}^{\ell q} &\equiv \eta_{RL}^{\ell q} - \eta_{LL}^{\ell q}. \qquad (25)
\end{aligned}$$

As it follows from Eq. (2), one has six independent parameters

$$\begin{aligned}
\eta_{AA}^{eu} &= \frac{g_{Z'}^2 Y_\phi^2}{4m_{Z'}^2}, \quad \eta_{LA}^{\ell u} = -\frac{g_{Z'}^2 Y_\ell^L Y_\phi}{2m_{Z'}^2}, \\
\eta_{AL}^{eu} &= \frac{g_{Z'}^2 Y_u^L Y_\phi}{2m_{Z'}^2}, \quad \eta_{LL}^{\ell u} = -\frac{g_{Z'}^2 Y_\ell^L Y_u^L}{m_{Z'}^2}, \qquad (26)
\end{aligned}$$

which can be used as the basis.

The experiment constrains the specific linear combinations of the variables $\eta_{\lambda\xi}^{\ell q}$ (see Ref. [6]). Introducing the normalized couplings:

$$\begin{aligned}
\Delta C_{1q}^\ell &= -\frac{1}{2\sqrt{2}G_F}\left(\eta_{RR}^{\ell q} - \eta_{LR}^{\ell q} + \eta_{RL}^{\ell q} - \eta_{LL}^{\ell q}\right), \\
\Delta C_{2q}^\ell &= -\frac{1}{2\sqrt{2}G_F}\left(\eta_{RR}^{\ell q} + \eta_{LR}^{\ell q} - \eta_{RL}^{\ell q} - \eta_{LL}^{\ell q}\right), \\
\Delta C_{3q}^\ell &= -\frac{1}{2\sqrt{2}G_F}\left(\eta_{RR}^{\ell q} - \eta_{LR}^{\ell q} - \eta_{RL}^{\ell q} + \eta_{LL}^{\ell q}\right), \\
\Delta q_L &= -\frac{1}{2\sqrt{2}G_F}\eta_{LL}^{\nu_\mu q}, \\
\Delta q_R &= -\frac{1}{2\sqrt{2}G_F}\eta_{LR}^{\nu_\mu q}, \qquad (27)
\end{aligned}$$

where G_F is the Fermi constant, one can write down the experimental bounds as follows:

$$\begin{aligned}
&2\Delta C_{1u}^e - \Delta C_{1d}^e = 0.217 \pm 0.26; \\
&2\Delta C_{2u}^e - \Delta C_{2d}^e = -0.765 \pm 1.23; \qquad (28)
\end{aligned}$$

$$2\Delta C^{\mu}_{3u} - \Delta C^{\mu}_{3d} = -1.51 \pm 4.9;$$
$$2\Delta C^{\mu}_{2u} - \Delta C^{\mu}_{2d} = 1.74 \pm 6.3; \quad (29)$$
$$\Delta C^{e}_{1u} + \Delta C^{e}_{1d} = 0.0152 \pm 0.033; \quad (30)$$
$$-2.73\Delta C^{e}_{1u} + 0.65\Delta C^{e}_{1d} - 2.19\Delta C^{e}_{2u} + 2.03\Delta C^{e}_{2d} = -0.065 \pm 0.19; \quad (31)$$
$$376\Delta C^{e}_{1u} + 422\Delta C^{e}_{1d} = 0.96 \pm 0.92;$$
$$572\Delta C^{e}_{1u} + 658\Delta C^{e}_{1d} = 1.58 \pm 4.2; \quad (32)$$
$$\Delta u_L = -0.0032 \pm 0.0169;$$
$$\Delta u_R = -0.0084 \pm 0.0251;$$
$$\Delta d_L = 0.002 \pm 0.0136;$$
$$\Delta d_R = -0.0109 \pm 0.0631. \quad (33)$$

Eqs. (28)-(33) represent the results of SLAC $e-D$, CERN $\mu - C$, Bates $e - C$, Mainz $e - Be$, the atomic parity violation and the ν_μ-nucleon scattering experiments, respectively [6]. They determine the allowed region in the space of η- parameters.

By employing Eqs. (25), (28)-(33) it is easy to obtain the bounds on the quantities (26):

$$\begin{aligned} 0 &< \eta^{eu}_{AA} < 0.114\mathrm{TeV}^{-2}, \\ -0.018\mathrm{TeV}^{-2} &< \eta^{eu}_{AL} < 0.006\mathrm{TeV}^{-2}, \\ -0.437\mathrm{TeV}^{-2} &< \eta^{\mu u}_{LA} < 0.661\mathrm{TeV}^{-2}, \\ -0.667\mathrm{TeV}^{-2} &< \eta^{eu}_{LA} < 0.238\mathrm{TeV}^{-2}, \\ -0.423\mathrm{TeV}^{-2} &< \eta^{\mu u}_{LL} < 0.358\mathrm{TeV}^{-2} \end{aligned} \quad (34)$$

The first of the relations (34) gives possibility to determine the allowed magnitude of the observables $\Delta\sigma_\ell(Z^*)$ and $\Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*)$

$$\begin{aligned} &-0.13\mathrm{pb} < \Delta\sigma_\ell(Z^*) < 0, \\ &-0.32\mathrm{pb} < \Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*) < 0. \end{aligned} \quad (35)$$

Thus, the signals of the abelian Z' must respect the above relations if the low energy physics is described by the minimal SM.

It is worth to compare the first of Eqs. (35) based on the analysis of the lepton-quark interactions with the direct constraints derived from the experiments on the $e^+e^- \to \ell^+\ell^-$ scattering. Introducing the normalized Z' coupling to the axial leptonic current:

$$|A_\ell| = \sqrt{\frac{g^2_{Z'} Y^{a2}_\ell m^2_Z}{4\pi m^2_{Z'}}}, \quad (36)$$

one can write down the bounds obtained from the processes $e^+e^- \to \ell^+\ell^-$ as follows $|A_\ell| < 0.025$ (see Fig. 2.7 of Ref. [1]). However, the relations (35) lead to the constraint $|A_\ell| < 0.0087$. Thus, the bounds (35) are about nine times stronger than ones derived from the analysis of the pure leptonic interactions.

V. DISCUSSION

In the lack of reliable information on the model describing physics beyond the SM and predicting Z' boson it is of great importance to find the model independent variables to search for this particle. In this regard, it could be useful to employ the method of Ref. [4] which gives possibility to reduce the number of unknown numbers parameterizing effects on new heavy virtual particles.

In Ref. [2] the observables $\sigma_\pm$ alternative to the familiar σ_T and A_{FB} and perspective in searching for the abelian Z' signal in the leptonic processes $e^+e^- \to \ell^+\ell^-$ were proposed. It was pointed out that the sign of the σ_+ deviation from the SM value can be uniquely predicted when the lepton universality is assumed. The sign remains to be unspecified in the case of an arbitrary interaction of Z' with leptons.

The observables introduced in the present paper are the extension of that in Ref. [2] with the specified above choice of the boundary angle. The observable $\Delta\sigma_\ell(Z^*)$ is the negative defined quantity even in the case when the lepton universality is not assumed. Moreover, the observable $\Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*)$ constructed for the quarks of the same generation is proportional to the leptonic one $\Delta\sigma_\ell(Z^*)$ being negative, too.

As it was mentioned in section III, the magnitude of the leptonic observable $\Delta\sigma_\ell(Z^*)$ is determined by means of the ρ parameter. Thus, the Z' contributions to the quantities $1 - \rho = 1 - m^2_W/m^2_Z \cos^2\theta_W$, $\Delta\sigma_\ell(Z^*)$ and $\Delta\sigma_{q_u}(Z^*) + 0.5\Delta\sigma_{q_d}(Z^*)$ have to be negative allowing to detect the Z' signal.

As the corollary of our analysis we note that the introduced observables allow one to identify (or to discard) the abelian Z' effects when the low energy physics is described by the minimal SM. Therefore, they can usefully complement the conventional analysis of Z' couplings based on the observables σ_T and A_{FB}.

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TABLE I. Energy dependence of $F_i^f(Z^*)$.

$\sqrt{s}$,GeV	500	600	700
Z^*	0.3170	0.3144	0.3129
$F_0^\ell(Z^*)$	-0.8012	-0.7889	-0.7815
$F_2^\ell(Z^*)$	0.0346	0.0341	0.0338
$F_3^\ell(Z^*)$	-0.0346	-0.0341	-0.0338
$F_0^{q_u}(Z^*)$	-0.5277	-0.5216	-0.5179
$F_1^{q_u}(Z^*)$	0.4250	0.4215	0.4194
$F_2^{q_u}(Z^*)$	-0.2532	-0.2499	-0.2479
$F_3^{q_u}(Z^*)$	-0.0331	-0.0296	-0.0276
$F_0^{q_d}(Z^*)$	-0.2513	-0.2522	-0.2527
$F_1^{q_d}(Z^*)$	0.8500	0.8430	0.8388
$F_2^{q_d}(Z^*)$	-0.5410	-0.5339	-0.5297
$F_3^{q_d}(Z^*)$	-0.0362	-0.0282	-0.0235

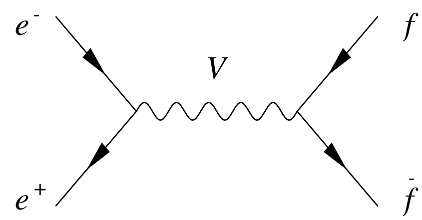

FIG. 1. The amplitude T_V of the process $e^+e^- \to V^* \to \bar{f}f$ at the Born level.

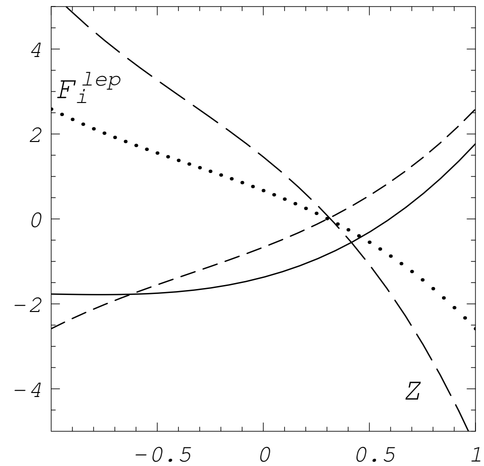

FIG. 2. The leptonic functions F_0^ℓ (the solid curve), F_1^ℓ (the long-dashed curve), F_2^ℓ (the dashed curve) and F_3^ℓ (the dotted curve) at $\sqrt{s} = 500$GeV.

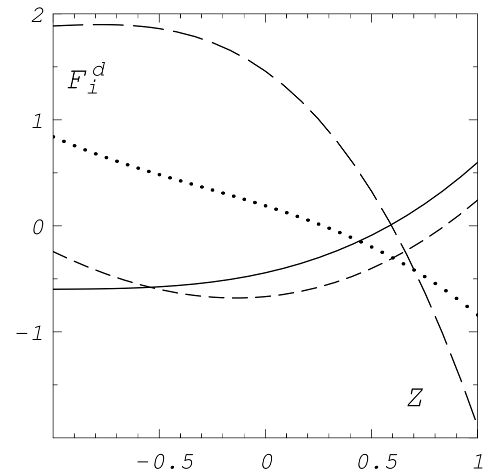

FIG. 4. The quark functions ($q_d = d, s, b$) $F_0^{q_d}$ (the solid curve), $F_1^{q_d}$ (the long-dashed curve), $F_2^{q_d}$ (the dashed curve) and $F_3^{q_d}$ (the dotted curve) at $\sqrt{s} = 500$GeV.

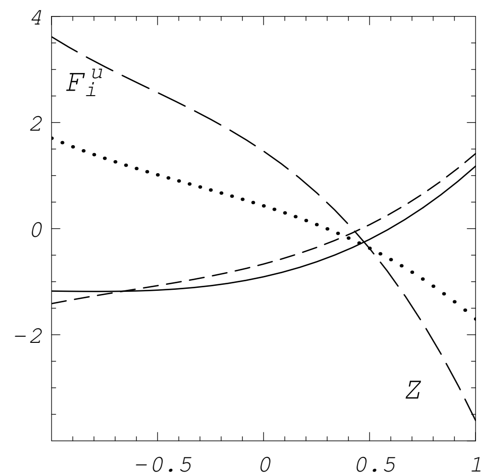

FIG. 3. The quark functions ($q_u = u, c$) $F_0^{q_u}$ (the solid curve), $F_1^{q_u}$ (the long-dashed curve), $F_2^{q_u}$ (the dashed curve) and $F_3^{q_u}$ (the dotted curve) at $\sqrt{s} = 500$GeV.

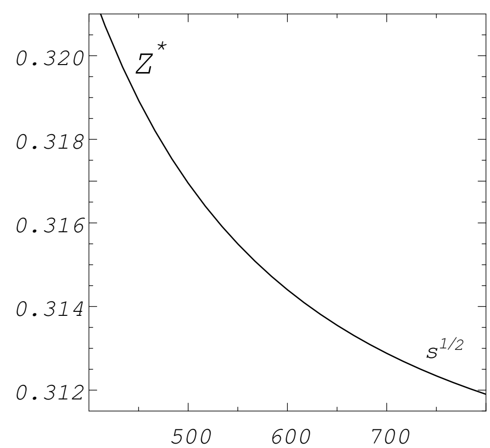

FIG. 5. Z^* as the function of $\sqrt{s}$(GeV).